Quantifying Trader Beliefs Through Yield Deviation: An Evolutionary Approach

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Abstract

A Markowitz efficient portfolio suggests that it is possible to maximize return given a specific level of risk. In this paper we utilize a model developed by Brock and Hommes in order to quantify trader beliefs through yield deviation. In the model we account for four types of traders: Fundamentalists, Perfect Foresighted, Trend Chasers and Contrarians. We calculate profit by subtracting the risk free rate *daily 3 month t-bill yield* from the risky rate *daily S&P 500 index yield* for a period of 33 years. Utilizing replicator dynamics we find convergence for share of the population in scenarios allocated between traders of perfect foresight and trend chasing, fundamentalist and trend chasing, & fundamentalist and contrarian.

1 Introduction

A Markowitz efficient portfolio is a portfolio that yields the highest possible return such that risk is limited to a desired amount. In other words, only allowing more risk into the portfolio can increase the expected return. Markowitz's formalization of diversification laid the foundation for the creation of the capital asset pricing model. The model is used to determine minimum rates of return for an asset and is the primary tool in making decisions about which assets should be included in a portfolio. The model has become the best approach in achieving the optimal level of diversification in a portfolio, *given some degree of risk*. This model was developed decades ago and since then it has been considered as a canonical model in finance literature.

The CAPM model allows for unsystematic risk to be diversified away. Essentially the model accounts for the specific risk of any given firm. The equation for market return in this model is $r_a = r_f + \beta(r_m - r_f)$ Where the level of specific risk associated with the firm is captured in the parameter β. This parameter includes the risk of new competitors, regulatory changes, and management changes. However, systematic risk is unavoidable even when an investor holds the optimal market portfolio. The effects of systematic risk was most recently demonstrated during the 2008 financial crisis, but also during the dot com bubble and during the oil glut of the 1980's. We collected daily data from United States 3 Month Treasury Bill Yields and the United States S&P 500 index yields from January-1984 to December-2016. We calculate profit by subtracting the risk free rate *daily 3 month t-bill yield* from the daily S&P 500 index yields. This deviation value can be positive or negative. After finding the daily deviation we find the monthly deviation in order to narrow our focus to 396 months or periods.

It is understood that investors are able to to reap the benefit of rising prices by means of dividends. Because dividends are the primary measure for determining the intrinsic value of a stock we plan to allow for noise in our fundamental price through the dividend parameter $\bar{y}/(R-1)$ rather than using a constant dividend. We do this because some of the inflation in stock prices are the effects of irrational investing. For the purpose of our simulation we define four different trader types: Fundamentalists, Perfect Foresighted, Trend Chasers and Contrarians. Utilizing replicator dynamics we hope to find convergence for share of the population in scenarios allocated between traders of perfect foresight and trend chasing, fundamentalist and trend chasing, and fundamentalist and contrarian. Fundamentalists strongly believe in the efficient market hypothesis and do not bother with technical analysis. All three of the other trader types we define are considered technical analysts. Perfect foresight traders know exactly what the price will be in the next period, but they must pay a cost in order to acquire the information. We include this cost as a parameter in our model. Trend chasers buy when prices are rising and sell when prices are decreasing. These types of traders tend to immensely exceed their liquid capital by leveraging highly in the hope that prices will continue to rise. Although we don't include a parameter for leverage in the model. Contrarians are very similar to trend chasers, except that when prices are rising they sell, and as prices are decreasing they buy.

It is important to note that trading beliefs can sometimes lead to a trader holding a sub-optimal portfolio. On the contrary, rational traders will sell off an overvalued stock, which will edge prices back down to the intrinsic value. Rational traders demand for undervalued stocks is limited by way of speculative risk for future growth and the possibility of in progress mis-pricing. The fundamental risk involved results in a demand curve that is inelastic. However, systematic risk does not affect diversification because systematic risk affects both sub-optimal and optimal portfolios. In other words, whether a trader is rational or irrational, they are not shielded from loss in fundamental value of the assets in their portfolios when the entire market experiences a shock. We plan to utilize a replicator dynamic simulation in hopes to find convergence for shares of the population in scenarios allocated between traders of perfect foresight and trend chasing, fundamentalist and trend chasing, & fundamentalist and contrarian. Upon discovery of how the population will converge we plan to adjust the parameters in order to implement shocks to our model so that we can compare to real economic shocks from our data.

2 Motivation

Despite the large use of the CAPM, it has limitations that have spawned research and production of several papers. This theme is very broad, and the resulting literature is more extensive than something that we will be able to classify or systematize. Therefore, we have decided to focus on specific aspects of the financial literature that have not used the evolutionary game theory approach. One of these aspects of research is the consideration of variance and co-variance matrices. However, implementing the correlation into the simulation was more difficult in practice then we had anticipated.

In our second attempt of simulation we decided to utilize the risk premium yield from our acquired data. Our profit is determined by taking the yield of the S&P 500 index and subtracting the yield of the 3 month T-bill rate. We use the Brock and Hommes (1998) model to establish the relationship to returns according to four belief types.

If profit increases or decreases then a rational investor should re-balance their portfolio. In each of our paired scenarios we have different expectations. In the scenario with traders with perfect foresight and trend chasing traders we expect equal shares of the two populations. This is because if profit continually increases, the shares of the population will be split evenly because both traders with perfect foresight and trend chasers will want to buy. However, if an entire population wants to buy, there is no one to sell. Similarly, if profit decreases we expect both traders with perfect foresight and trend chasers will want to sell. However, if an entire population wants to sell, there is no one to buy. In the scenario with fundamentalist traders and trend chasing traders we expect more movement in the corresponding shares of the population. This is because if profit continually increases the share of the population will change because fundamentalist traders will hold or want to sell, whereas trend chasing traders will want to buy. Similarly, if profit decreases fundamentalist traders will hold or want to buy, whereas trend chasing traders will want to sell. In the scenario with fundamentalist traders and contrarian traders we expect less movement in the corresponding shares of the population. This is because if profit continually increases the share of the population will change because fundamentalist traders will hold or want to sell, while contrarian traders will also want to sell. If an entire population wants to sell, there is no one to buy. Similarly, if profit decreases fundamentalist traders will hold or want to buy, while contrarian traders will also want to buy. If an entire population wants to buy, there is no one to sell. We hope that through our simulation of replicator dynamics we will be able to find a basin of attraction for each paired scenario.

3 Literature Review

While we are not generating data from a computer-simulated financial market, it is worthwhile to look at models previously analyzed. Many of the relevant papers describe a large heterogeneous system which approximates evolutionary dynamics with many types of traders. Namely, Brock and Homes (1997, 1998) have done much work on this topic [BH97] [BH98]. In the 1998 paper, they observe the evolution of four different types of trader populations: fundamentalist, perfect foresight, trend chasing, and contrarians. We will be using their models to measure the relationship between these belief types and their returns. Brock, Homes, and Wagener (2005) later introduce the notion of a large-type limit system which is a much more heterogeneous, multidimensional system which is evolutionarily adaptive [BHW05]. Hence, it can be used to analyze stability and see how dynamics may differ from other models. While we will be using only four trading strategies, this paper helps understand price evolution.

Learning is also a topic that we may want to visit in the future. At the moment, traders in our model only respond to the behavior of other traders but we might eventually want them to respond to a lot of other types of exogenous news such as changes in the interest rate, industry trends, and perhaps the discovery of new trading strategy. Our data spans over a considerable period of time, in which new financial assets were being innovated at an increasingly growing rate. The implications of a lot of these trades were not understood because they were such a novel concept. LeBaron, Arthur, and Palmer (1999) talk about learning in a simulated financial market [LAP99]. Because equilibrium is not always reached in the market, strategies are constantly reevaluated and adapted. They also address variable selection to the agent's forecasting problem. Instead of taking into consideration all information in a given situation, agents have to decide what is relevant and then use that smaller portion to make their forecasts. This model does not take into account heterogeneity - the market is not made up of different strategy types. All agents are assumed to be the same type of investor and therefore maximize the same expected utility function.

4 Model

We followed Brock and Hommes (1998) paper on "Heterogeneous beliefs and routes to chaos in a simple asset pricing model". The idea here is to extend the evolutionary approach to get some behavior aspect over the prices. The way that the authors developed consisted in looking over a model of Adaptive Beliefs System (ABS) and Pricing Discounted Value (PDV) pricing model. The idea of a portfolio with a risk free asset and some risky assets imply that:

$$W_{t+1} = RW_t + (p_{t+1} + y_{t+1} - Rp_t)z_t \tag{1}$$

where R is the gross return of the risk free asset, W_t is the wealth, p_t is the price and y_{t+1} is the stochastic process of the risky asset's dividend.

Each Investor type is a mean variance maximizer, i.e., solves the following problem:

$$Max_z\{E_{ht}W_{t+1} - (a/2)V_{ht}(W_{t+1})\} \tag{2}$$

The solution of this maximization problem is found in the closed form

$$z_{ht} = \{E_{ht}(p_{t+1} + y_{t+1} - Rp_t)/a\sigma^2\}, \tag{3}$$

where "a" denotes the risk aversion, which is assumed to be equal for all traders. Also, it is assumed homogeneous and constant beliefs on variances - $V_{ht}(p_{t+1} + y_{t+1} - Rp_t) = \sigma^2$ for all types h.

Considering z_{st} the supply of shares per investor and n_{ht} the fraction of investors of type h at date t, equilibrium of demand and supply implies:

$$\sum n_{ht}\{E_{ht}(p_{t+1} + y_{t+1} - Rp_t)/a\sigma^2\} = z_{st} \tag{4}$$

With only one type h, market equilibrium yields the pricing equation,

$$Rp_t = E_{ht}(p_{t+1} + y_{t+1}) - a\sigma^2 z_{st} \tag{5}$$

Within the special case of zero supply outside shares ($z_{st} = 0$) and the "fundamental" solution as $Rp_t^* = E_t p_{t+1}^* + y_{t+1}$, $p^* = \bar{p}$ that solves $R\bar{p} = \bar{p} + \bar{y} \Rightarrow \bar{p} = \bar{y}/(R-1)$. That is the fundamental pricing of the asset.

The authors also assumed as convenient working with the deviation x_t from the benchmark fundamental p_t^*:

$$x_t = p_t - p_t^* \tag{6}$$

Rewriting for no outside shares, i.e., $z_{st} = 0$, we have,

$$Rp_t = \sum n_{ht} E_{ht}(p_{t+1} + y_{t+1}) \tag{7}$$

A big assumption of the model is that all beliefs are of the form

$$E_{ht}(p_{t+1} + y_{t+1}) = E_t(p_{t+1}^* + y_{t+1}) + f_h(x_{t-1}, \ldots, x_{t-L}) \tag{8}$$

where p_{t+1}^* denotes the fundamental, $E_t(p_{t+1}^* + y_{t+1})$ is the conditional expectation of the fundamental, $x_t = p_t - p_t^*$ is the deviation from the fundamental, and f_h is some deterministic function which can differ across trader types h. Manipulating the equations allows us to get

$$Rx_t = \sum n_{h,t-1} f_h(x_{t-1}, \ldots, x_{t-L}) = \sum n_{h,t-1} f_{ht} \tag{9}$$

The fitness function in this model is given by realized profits and defined as

$$\pi_{h.t} = R_{t+1} z(\rho_{ht}) = (x_{t+1} - Rx_t) z(\rho_{ht}) \tag{10}$$

It is also possible to extend it adding some memory to the fitness function as a performance measure.

$$U_{h,t} = \pi_{h,t} + \eta U_{h,t-1} \tag{11}$$

The last step in this section is to show the updating fractions $n_{h,t}$ to be given by the logistics form on the discrete choice probability.

$$\frac{exp[\beta U_{h,t-1}]}{\sum exp[\beta U_{h,t-1}]} \tag{12}$$

From now on, the belief type assumed in a functional form is plugged in the equations above to get Returns and Shares Coevolution process through time.

Assuming the belief types written over the form:

$$f_{ht} = g_h x_{t-1} + b_h, \tag{13}$$

where g_h is the trend and b_h is the bias of trader type h. If $b_h = 0$, h is a pure trend chaser if $g > 0$ (strong trend chaser if $g > R$) and a contrarian if $g < 0$ (strong contrarian if $g < R$). If $g_h = 0$, type h is said to be purely biased (upward biased if $b_h > 0$ and downward biased if $b_h < 0$).

In the special case $g_h = b_h = 0$, equation (13) reduces to fundamentalists, believing that prices return to their fundamental value. Fundamentalists do have all past prices and dividends in their information set, but they do not know the fractions $n_{h,t}$ of the other belief types.

Rational agents have perfect foresight. At each date they know not only all past prices and dividends, but also the market equilibrium equation, with all fractions $n_{h,t}$ of other belief types. Rational agents are thus able to compute x_{t+1} perfectly.

Combining the equations we have:

$$\pi_{j,t-1} = \frac{1}{a\sigma^2}(x_t - Rx_{t-1})(g_h x_{t-2} + b_h - Rx_{t-1}), \tag{14}$$

for a type described as equation (13), and

$$\pi_{R,t-1} = \frac{1}{a\sigma^2}(x_t - Rx_{t-1})^2 - C, \tag{15}$$

for rational agents where $C \geq 0$ are costs for rational expectations. Accumulated past profits are:

$$U_{j,t-1} = \pi_{j,t-1} + \eta U_{j,t-2}, \;\; 0 \leq \eta \leq 1 \tag{16}$$

We will apply three comparisons between two different types of investors. The first scenario described is "Perfect foresight versus trend chaser". From equation (9), the return is written as:

$$Rx_t = n_{1,t-1}x_{t+1} + n_{2,t-1}gx_{t-1} \tag{17}$$

Update fractions use equations (14) - (16) to be defined:

$$n_{1,t} = exp[\beta(\frac{1}{a\sigma^2}(x_t - Rx_{t-1})^2 + \eta U_{1,t-2} - C)]/z_t \tag{18}$$
$$n_{2,t} = exp[\beta(\frac{1}{a\sigma^2}(x_t - Rx_{t-1})(gx_{t-2} - Rx_{t-1}) + \eta U_{2,t-2})]/z_t \tag{19}$$

where $z_t = \sum exp[\beta U_{h,t-1}]$

Applying this framework to a second scenario, "Fundamentalists versus trend chasers", we obtain:

$$Rx_t = n_{2,t-1}gx_{t-1} \tag{20}$$

Update fractions at this scenario will be defined as'

$$n_{1,t} = exp[\beta(\frac{1}{a\sigma^2}Rx_{t-1}(Rx_{t-1} - x_t) - C)]/z_t \quad (21)$$

$$n_{2,t} = exp[\beta(\frac{1}{a\sigma^2}(x_t - Rx_{t-1})(gx_{t-2} - Rx_{t-1}))]/z_t \quad (22)$$

Looking for a scenario with "Fundamentalists versus contrarians", the adaptive belief system is identical as the one composed by equations (20) - (22), with $g < 0$ as the only difference.

4.1 Data

We collected daily data from January 1984 to December 2016 for T-Bills with 90 days of maturity and S&P500 Index. As daily data has a lot of noise and this data set contains 396 months, aggregating the data monthly don't compromise our analysis. We also treated the data to calculate the monthly returns of the two instruments. With this information we are able to calculate deviations between risky asset (S&P500) and risk-free asset (T-Bill 90 days), and apply the framework from Brock and Hommes described in the section above with the different beliefs.

4.2 Simulations

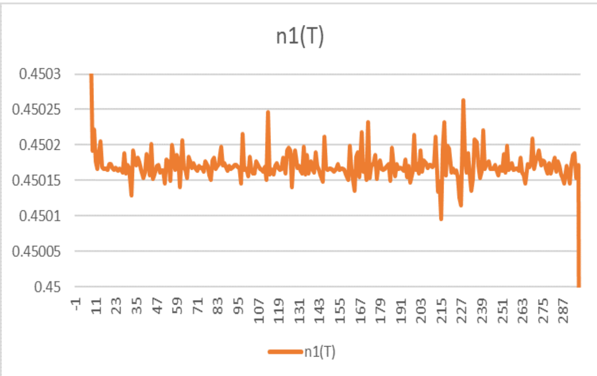
As described above, the simulations rely over three comparison. The theoretical equations of deviations from fundamental price and shares were derived and generated the results described below. First, the auxiliar parameters are described (PF $\rightarrow$ perfect foresight, TC $\rightarrow$ trend chaser, Fund $\rightarrow$ Fundamentalist, Contr $\rightarrow$ Contrarians).

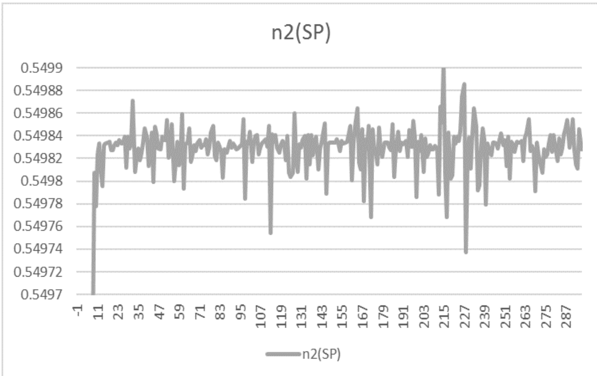
Parameters	PF x TC	Fund x TC	Fund x Contr
β	1	0.5	0.5
a	1	1	1
σ	4	4	4
η	0.9	0.8	0.6
C	0.2	2	0.2
g	0.7	0.9	-0.7

We simulated with three different initial share distributions: 50% for each asset, 80% for risk-free asset and 20% for the risky asset, and 20% for the risk-free asset and 80% for the risky one.

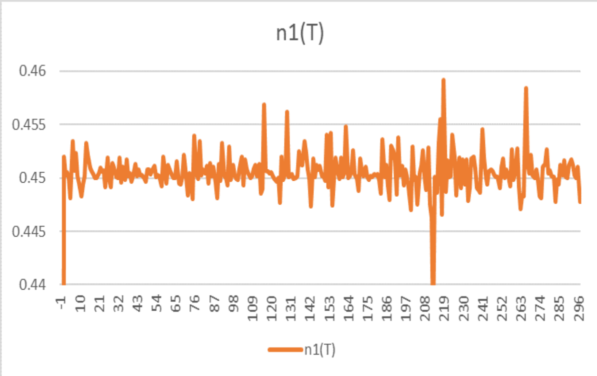
4.2.1 Perfect Foresight x Trend Chaser

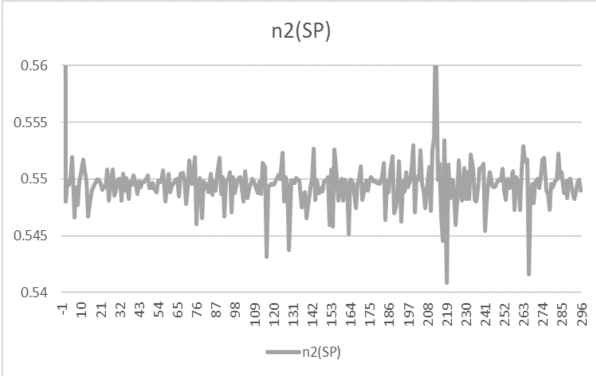
t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	0.44	0.50	3	3	0.5	0.5	0.5	0.5
0	0.48576	0.48854	3.20000	3.20000	0.5	0.5	0.50	0.50
1	0.60005	0.58386	3.38373	3.37627	0.44640	0.55360	0.52	0.48
2	0.58917	0.55427	3.56233	3.52167	0.45260	0.54740	0.51	0.49
3	0.36541	0.36597	3.71818	3.65742	0.46082	0.53918	0.50	0.50
4	0.24372	0.24331	3.84500	3.79304	0.45276	0.54724	0.52	0.48
5	0.07175	0.07057	3.97610	3.89814	0.45265	0.54735	0.50	0.50
6	(0.06680)	(0.06377)	4.07853	4.00829	0.44994	0.55006	0.52	0.48
7	(0.04221)	(0.04132)	4.18653	4.09160	0.45045	0.54955	0.48	0.52



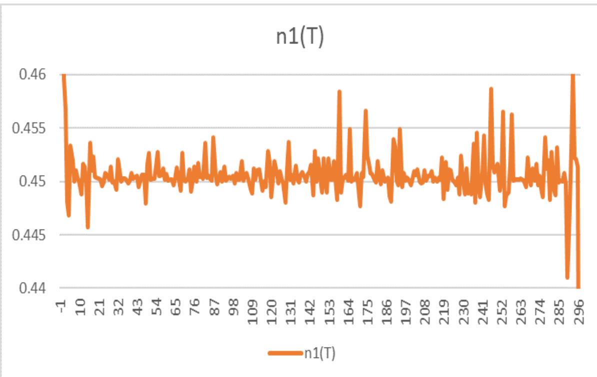


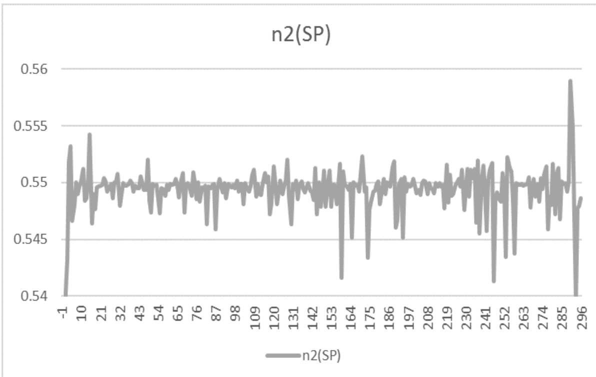
t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	0.26	0.50	3	3	0.2	0.8	0.5	0.5
0	0.28566	0.28730	3.20000	3.20000	0.2	0.8	0.50	0.50
1	0.15967	0.15536	3.38373	3.37627	0.45195	0.54805	0.52	0.48
2	0.00893	0.00840	3.56233	3.52167	0.45127	0.54873	0.51	0.49
3	0.13807	0.13828	3.71818	3.65742	0.45089	0.54911	0.50	0.50
4	0.27073	0.27028	3.84500	3.79304	0.44796	0.55204	0.52	0.48
5	0.20806	0.20463	3.97610	3.89814	0.45199	0.54801	0.50	0.50
6	0.25136	0.23996	4.07853	4.00829	0.44972	0.55028	0.52	0.48
7	0.20925	0.20482	4.18653	4.09160	0.45120	0.54880	0.48	0.52





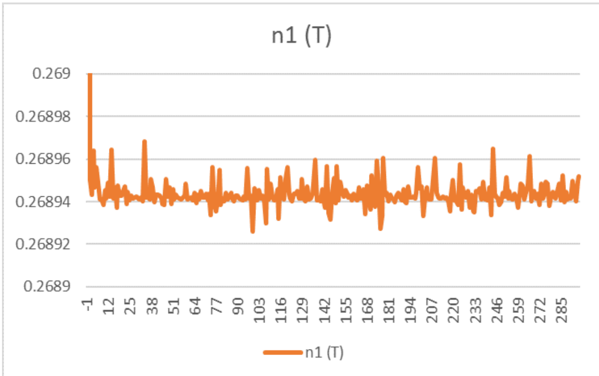
t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	0.36	0.50	3	3	0.8	0.2	0.5	0.5
0	0.40017	0.40246	3.20000	3.20000	0.8	0.2	0.50	0.50
1	0.57692	0.56135	3.38373	3.37627	0.44568	0.55432	0.52	0.48
2	0.61267	0.57637	3.56233	3.52167	0.45020	0.54980	0.51	0.49
3	0.48218	0.48291	3.71818	3.65742	0.45686	0.54314	0.50	0.50
4	0.47908	0.47827	3.84500	3.79304	0.45030	0.54970	0.52	0.48
5	0.39421	0.38771	3.97610	3.89814	0.45367	0.54633	0.50	0.50
6	0.29809	0.28457	4.07853	4.00829	0.45325	0.54675	0.52	0.48

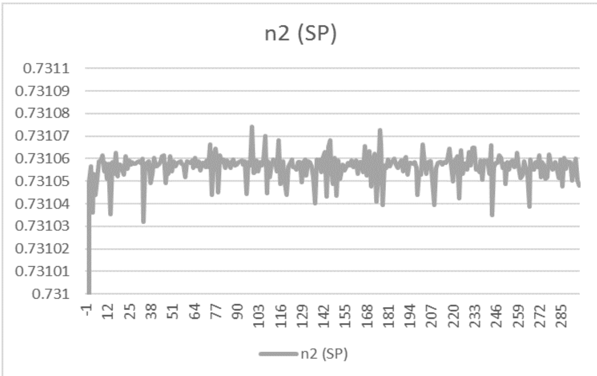




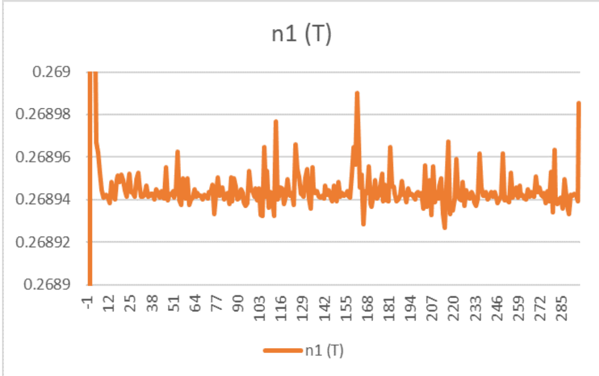
4.2.2 Fundamentalists x Trend Chaser

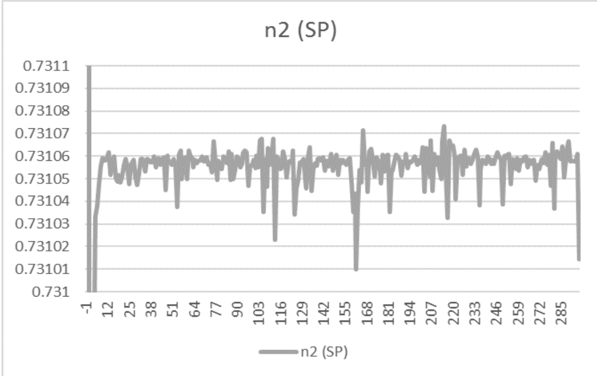
t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	0.21	0.50	4	4	0.5	0.5	0.5	0.5
0	0.23590	0.23726	3.70000	3.70000	0.5	0.5	0.50	0.50
1	0.11571	0.11259	3.46373	3.45627	0.26901	0.73099	0.52	0.48
2	0.06445	0.06063	3.28796	3.24804	0.26897	0.73103	0.51	0.49
3	0.02440	0.02444	3.14245	3.08635	0.26897	0.73103	0.50	0.50
4	0.00877	0.00876	3.01260	2.97044	0.26894	0.73106	0.52	0.48
5	0.00141	0.00138	2.92568	2.86076	0.26894	0.73106	0.50	0.50
6	0.01242	0.01185	2.84058	2.78857	0.26894	0.73106	0.52	0.48
7	(0.01072)	(0.01050)	2.78832	2.71500	0.26895	0.73105	0.48	0.52



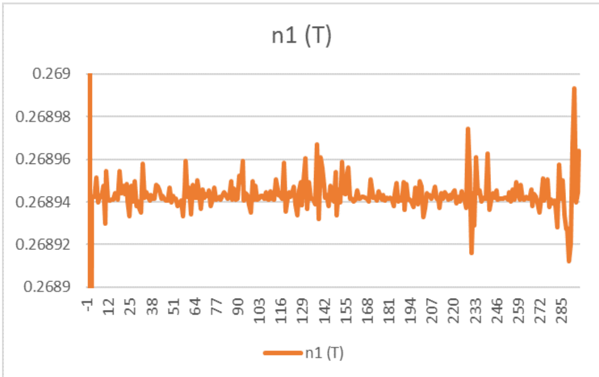


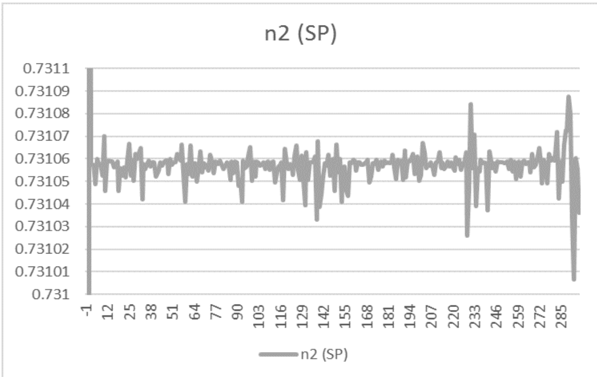
t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	0.32	0.50	4	4	0.2	0.8	0.5	0.5
0	0.35841	0.36046	3.70000	3.70000	0.2	0.8	0.50	0.50
1	0.27322	0.26584	3.46373	3.45627	0.26955	0.73045	0.52	0.48
2	0.20074	0.18885	3.28796	3.24804	0.26940	0.73060	0.51	0.49
3	0.11949	0.11968	3.14245	3.08635	0.26926	0.73074	0.50	0.50
4	0.07710	0.07697	3.01260	2.97044	0.26901	0.73099	0.52	0.48
5	0.05900	0.05802	2.92568	2.86076	0.26896	0.73104	0.50	0.50
6	0.00796	0.00760	2.84058	2.78857	0.26900	0.73100	0.52	0.48
7	0.00540	0.00529	2.78832	2.71500	0.26894	0.73106	0.48	0.52





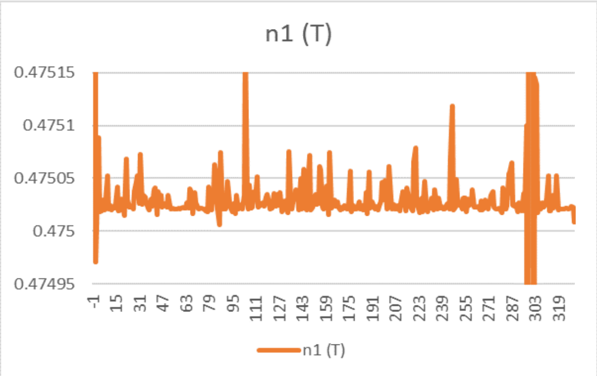
t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	0.07	0.50	4	4	0.8	0.2	0.5	0.5
0	0.08124	0.08171	3.70000	3.70000	0.8	0.2	0.50	0.50
1	0.01337	0.01301	3.46373	3.45627	0.26846	0.73154	0.52	0.48
2	(0.00284)	(0.00267)	3.28796	3.24804	0.26892	0.73108	0.51	0.49
3	(0.00297)	(0.00297)	3.14245	3.08635	0.26894	0.73106	0.50	0.50
4	0.00002	0.00002	3.01260	2.97044	0.26894	0.73106	0.52	0.48
5	0.01228	0.01208	2.92568	2.86076	0.26894	0.73106	0.50	0.50
6	0.01280	0.01221	2.84058	2.78857	0.26894	0.73106	0.52	0.48
7	0.00299	0.00293	2.78832	2.71500	0.26894	0.73106	0.48	0.52

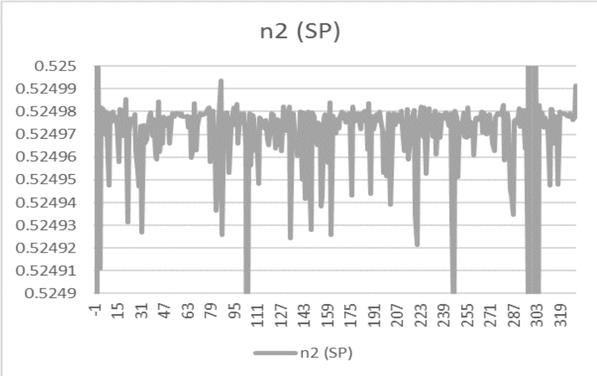




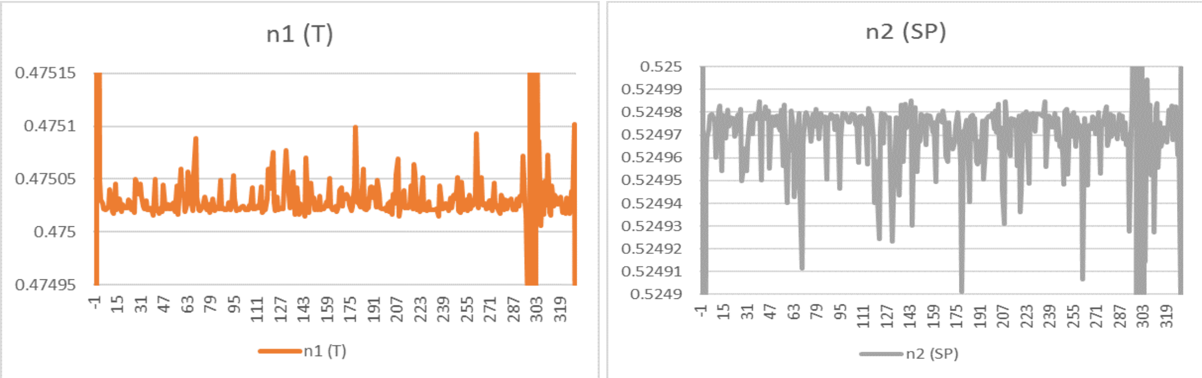
4.2.3 Fundamentalists x Contrarians

t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	-0.15151	0.5	4	4	0.5	0.5	0.5	0.5
0	-0.16834	-0.16931	2.9	2.9	0.5	0.5	0.50373	0.49627
1	0.06876	0.0669	2.24373	2.23627	0.47492	0.52508	0.51697	0.48303
2	-0.01482	-0.01394	1.86321	1.82479	0.47507	0.52493	0.51208	0.48792
3	0.00383	0.00384	1.63001	1.58279	0.47501	0.52499	0.49864	0.50136
4	0.01079	0.01077	1.47664	1.45104	0.47502	0.52498	0.5156	0.4844
5	0.01914	0.01882	1.40159	1.35502	0.47501	0.52499	0.50004	0.49996
6	-0.00535	-0.00511	1.34099	1.31298	0.47506	0.52494	0.51586	0.48414
7	-0.01035	-0.01013	1.32045	1.27193	0.47502	0.52498	0.48047	0.51953

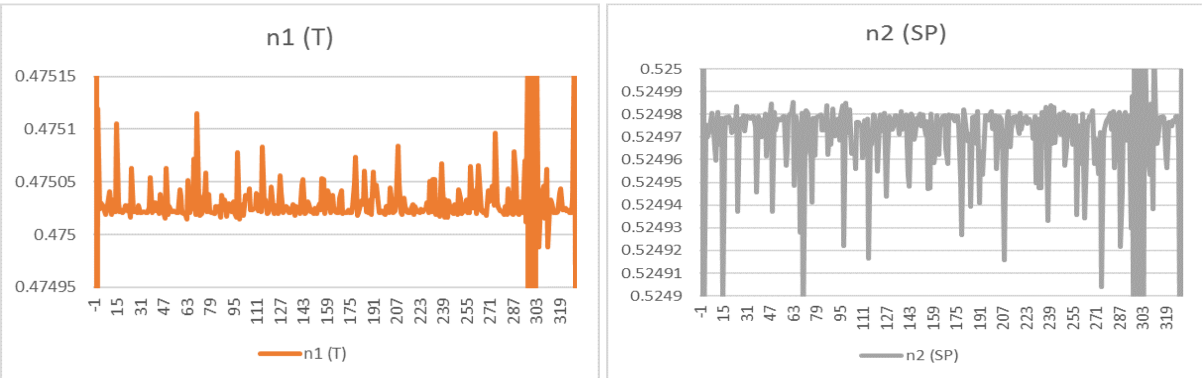




t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	-0.24212	0.5	4	4	0.2	0.8	0.5	0.5
0	-0.26903	-0.27057	2.9	2.9	0.2	0.8	0.50373	0.49626
1	0.16095	0.15661	2.24373	2.23626	0.47751	0.52248	0.51697	0.48302
2	-0.05312	-0.04997	1.86321	1.82478	0.47589	0.5241	0.51208	0.48791
3	0.00818	0.00819	1.63	1.58279	0.47501	0.52498	0.49863	0.50136
4	-0.00766	-0.00764	1.47664	1.45103	0.47501	0.52498	0.51559	0.4844
5	-0.00825	-0.00811	1.40158	1.35502	0.47502	0.52497	0.50003	0.49996
6	-0.00395	-0.00377	1.34098	1.31297	0.47502	0.52497	0.51585	0.48414
7	-0.0004	-0.00039	1.32044	1.27193	0.47502	0.52497	0.48046	0.51953



t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	(0.07)	0.50	4	4	0.8	0.2	0.5	0.5
0	(0.07471)	(0.07513)	2.90000	2.90000	0.5	0.5	0.50	0.50
1	0.01893	0.01842	2.24373	2.23627	0.47444	0.52556	0.52	0.48
2	(0.00635)	(0.00597)	1.86321	1.82479	0.47501	0.52499	0.51	0.49
3	0.00244	0.00244	1.63001	1.58279	0.47502	0.52498	0.50	0.50
4	(0.00173)	(0.00173)	1.47664	1.45104	0.47502	0.52498	0.52	0.48
5	0.00660	0.00649	1.40159	1.35502	0.47502	0.52498	0.50	0.50
6	(0.00725)	(0.00692)	1.34099	1.31298	0.47503	0.52497	0.52	0.48
7	0.01917	0.01877	1.32045	1.27193	0.47503	0.52497	0.48	0.52



5 Conclusion

We analyzed trading beliefs and based on the functional form of these beliefs we were able to find convergence in replicator dynamics for every paired scenario we simulated. In the scenario with traders with perfect foresight and trend chasing traders we expected convergence to equal shares of the population. The results came very close to our expectation. In the simulation with initial distribution being equal at .5 and .5 the shares of the population converged in the seventh period to .45 perfect foresight and .55 trend chaser. In the simulation with the initial distribution being .2 for perfect foresight and .8 for trend chaser the population converged in the seventh period to .45 perfect foresight and .55 trend chaser. In the simulation for .8 perfect foresight and .2 trend chaser the population converged in the sixth period to .45 perfect foresight and .55 trend chaser.

In the scenario with fundamentalist traders and trend chasing traders we expected more movement in the corresponding shares of the population. The results came very close to our expectation. In the simulation with initial distribution being equal at .5 and .5, the shares of the population converged in the seventh period to .27 fundamentalist and .73 trend chaser. In the simulation with the initial distribution being .2 for fundamentalist and .8 for trend chaser, the population converged in the seventh period to .27 fundamentalist and .73 trend chaser. In the simulation for .8 fundamentalist and .2 trend chaser, the population converged in the seventh period to .27 fundamentalist and .73 trend chaser.

In the scenario with fundamentalist traders and contrarian traders we expected less movement in the corresponding shares of the population. The results came very close to our expectation. In the simulation with initial distribution being equal at .5 and .5, the shares of the population converged in the seventh period to .48 fundamentalist and .52 contrarian. In the simulation with the initial distribution being .2 for fundamentalist and .8 for contrarian, the population converged in the seventh period to .48 fundamentalist and .52 contrarian. In the simulation for .8 fundamentalist and .2 contrarian, the population converged in the seventh period to .48 fundamentalist and .52 trend chaser.

In regards to future work, we would like to extend the model we utilized to mirror real data. Although this could be problematic due to the nature of economic shocks. We would like to examine the effects of such shocks through the inputs of our parameter values. We would also like to check the fitness of opposing strategies with CAPM. This would allow us to increase variable analysis and utilize correlation of assets in each simulation. Furthermore, we would like to run simulations utilizing foreign assets with simulations including the NIKKEI index from Japan.

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