Quantifying Trader Beliefs Through Yield Deviation: An Evolutionary Approach

F. Chertman
S. Sharma
W. Sump

University of California Santa Cruz

Department of Economics

March 31, 2017

Overview

Introduction

- Markowitz, Modern Portfolio Theory
- Risk is an inherent part of reward
- Suggests that it is possible to maximize return for a certain level of risk
- Formalized idea of diversification - risk-averse investors can minimize exposure to certain types of risk by investing in different assets

Introduction, CAPM Model

- CAPM model is crucial in creating the optimal portfolio and minimizing systematic risk
- $r_a = r_f + \beta(r_m - r_f)$
- Two types of risk: systematic and unsystematic
- Systematic risk cannot be diversified away even when holding the optimal market portfolio

Motivation

Despite the large use of the CAPM, it has limitations that have spawned research and production of several papers. This theme is very broad, and the resulting literature is more extensive than something that we will be able to classify or systematize. Therefore, we have decided to focus on specific aspects of the financial literature that have not used the evolutionary game theory approach. One of these aspects of research is the consideration of variance and co-variance matrices. However, implementing the correlation into the simulation was more difficult in practice then we had anticipated.

Application

In our second attempt of simulation we decided to utilize the risk premium yield from our acquired data. Our profit is determined by taking the yield of the S&P 500 index and subtracting the yield of the 3 month T-bill rate. We use the Brock and Hommes (1998) model to establish the relationship to returns according to some belief types.

Types of Traders

- Fundamentalists
- Perfect Foresight
- Trend Chaser
- Contrarian

Data

We collected daily data from United States T-Bill Yields, the United States S&P 500 index and the Japanese Nikkei Index from January-1984 to December-2016.

Model

- Adaptive beliefs in Price Discounted Value (PDV) asset pricing model
  - Dynamics of wealth
  - $W_{t+1} = RW_t + (p_{t+1} + y_{t+1} - Rp_t)z_t$
- Each Investor type is a mean variance maximizer, i.e., solves the following problem:
  - $Max_z\{E_{ht}W_{t+1} - (a/2)V_{ht}(W_{t+1})\}$
  - $z_{ht} = \{E_{ht}(p_{t+1} + y_{t+1} - Rp_t)/a\sigma^2\}$

Model

- Equilibrium of demand and supply
  - $\sum n_{ht}\{E_{ht}(p_{t+1} + y_{t+1} - Rp_t)/a\sigma^2\} = z_{st}$
- Market equilibrium yields the pricing equation
  - $Rp_t = E_{ht}(p_{t+1} + y_{t+1}) - a\sigma^2 z_{st}$
- Fundamental price with constant dividend
  - $R\bar{p} = \bar{p} + \bar{y} \Rightarrow \bar{p} = \bar{y}/(R-1)$
- Deviation from benchmark fundamental
  - $x_t = p_t - p_t^*$

Model

- Rewriting for no outside shares
  - $Rp_t = \sum n_{ht} E_{ht}(p_{t+1} + y_{t+1})$
- Beliefs are of the form
  - $E_{ht}(p_{t+1} + y_{t+1}) = E_t(p^*_{t+1} + y_{t+1}) + f_h(x_{t-1}, \ldots, x_{t-L})$
- Manipulating the equations
  - $Rx_t = \sum n_{h,t-1} f_h(x_{t-1}, \ldots, x_{t-L}) = \sum n_{h,t-1} f_{ht}$

Model

- Fitness function (given by realized profits)
  - $\pi_{h.t} = R_{t+1} z(\rho_{ht}) = (x_{t+1} - Rx_t) z(\rho_{ht})$
- Memory in the performance measure
  - $U_{h,t} = \pi_{h,t} + \eta U_{h,t-1}$
- Update fractions given by discrete choice probability
  - $\frac{exp[\beta U_{h,t-1}]}{\sum exp[\beta U_{h,t-1}]}$

Model

- Belief Types
  - $f_{ht} = g_h x_{t-1} + b_h$
- Perfect foresight versus trend chaser
  - $Rx_t = n_{1,t-1} x_{t+1} + n_{2,t-1} gxt - 1$
- Update fractions
  - $n_{1,t} = exp[\beta(\frac{1}{a\sigma^2}(x_t - Rx_{t-1})^2 + \eta U_{1,t-2} - C)]/z_t$
  - $n_{2,t} = exp[\beta(\frac{1}{a\sigma^2}(x_t - Rx_{t-1})(gx_{t-2} - Rx_{t-1}) + \eta U_{2,t-2})]/z_t$

Model

- Fundamentalists versus trend chasers
  - $Rx_t = n_{2,t-1} g x t - 1$
- Update fractions
  - $n_{1,t} = exp[\beta(\frac{1}{a\sigma^2} Rx_{t-1}(Rx_{t-1} - x_t) - C)]/z_t$
  - $n_{2,t} = exp[\beta(\frac{1}{a\sigma^2}(x_t - Rx_{t-1})(gx_{t-2} - Rx_{t-1}))]/z_t$
- Fundamentalists versus contrarians

Parameters

Parameters	PF x TC	Fund x TX	Fund x Contr
β	1	0.5	0.5
a	1	1	1
σ	4	4	4
η	0.9	0.8	0.6
C	0.2	2	0.2
g	0.7	0.9	-0.7

Perfect Foresight x Trend Chaser

t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	0.44	0.50	3	3	0.5	0.5	0.5	0.5
0	0.48576	0.48854	3.20000	3.20000	0.5	0.5	0.50	0.50
1	0.60005	0.58386	3.38373	3.37627	0.44640	0.55360	0.52	0.48
2	0.58917	0.55427	3.56233	3.52167	0.45260	0.54740	0.51	0.49
3	0.36541	0.36597	3.71818	3.65742	0.46082	0.53918	0.50	0.50
4	0.24372	0.24331	3.84500	3.79304	0.45276	0.54724	0.52	0.48
5	0.07175	0.07057	3.97610	3.89814	0.45265	0.54735	0.50	0.50
6	(0.06680)	(0.06377)	4.07853	4.00829	0.44994	0.55006	0.52	0.48
7	(0.04221)	(0.04132)	4.18653	4.09160	0.45045	0.54955	0.48	0.52

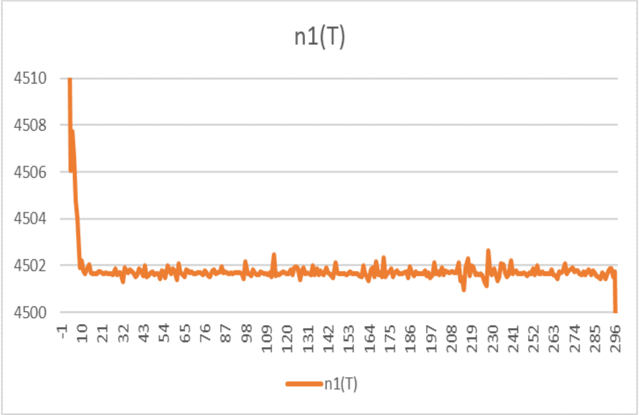
Fundamentalist x Trend Chaser

t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	0.32	0.50	4	4	0.2	0.8	0.5	0.5
0	0.35841	0.36046	3.70000	3.70000	0.2	0.8	0.50	0.50
1	0.27322	0.26584	3.46373	3.45627	0.26955	0.73045	0.52	0.48
2	0.20074	0.18885	3.28796	3.24804	0.26940	0.73060	0.51	0.49
3	0.11949	0.11968	3.14245	3.08635	0.26926	0.73074	0.50	0.50
4	0.07710	0.07697	3.01260	2.97044	0.26901	0.73099	0.52	0.48
5	0.05900	0.05802	2.92568	2.86076	0.26896	0.73104	0.50	0.50
6	0.00796	0.00760	2.84058	2.78857	0.26900	0.73100	0.52	0.48
7	0.00540	0.00529	2.78832	2.71500	0.26894	0.73106	0.48	0.52

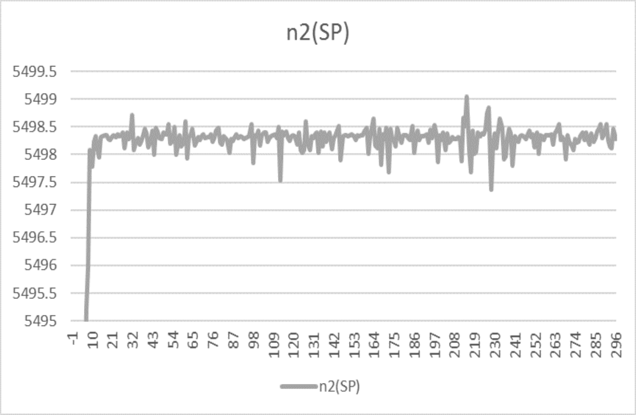
Fundamentalist x Contrarian

t	Rx_t	W_x (SP-T) $= x_t$	$U_{1,t}$	$U_{2,t}$	$n_1(T)$	$n_2(SP)$	Fitness 1	Fitness 2
-1	(0.07)	0.50	4	4	0.8	0.2	0.5	0.5
0	(0.07471)	(0.07513)	2.90000	2.90000	0.5	0.5	0.50	0.50
1	0.01893	0.01842	2.24373	2.23627	0.47444	0.52556	0.52	0.48
2	(0.00635)	(0.00597)	1.86321	1.82479	0.47501	0.52499	0.51	0.49
3	0.00244	0.00244	1.63001	1.58279	0.47502	0.52498	0.50	0.50
4	(0.00173)	(0.00173)	1.47664	1.45104	0.47502	0.52498	0.52	0.48
5	0.00660	0.00649	1.40159	1.35502	0.47502	0.52498	0.50	0.50
6	(0.00725)	(0.00692)	1.34099	1.31298	0.47503	0.52497	0.52	0.48
7	0.01917	0.01877	1.32045	1.27193	0.47503	0.52497	0.48	0.52

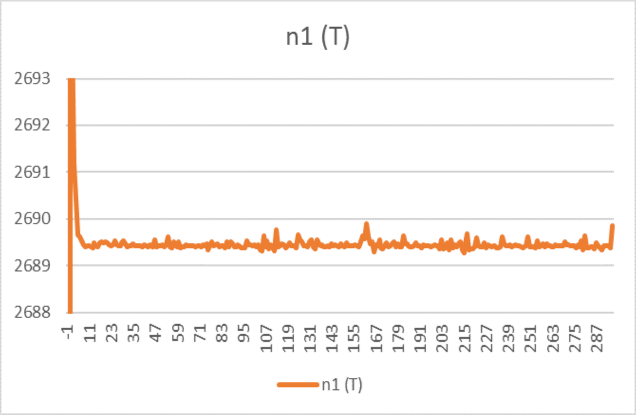
Perfect Foresight x Trend Chaser



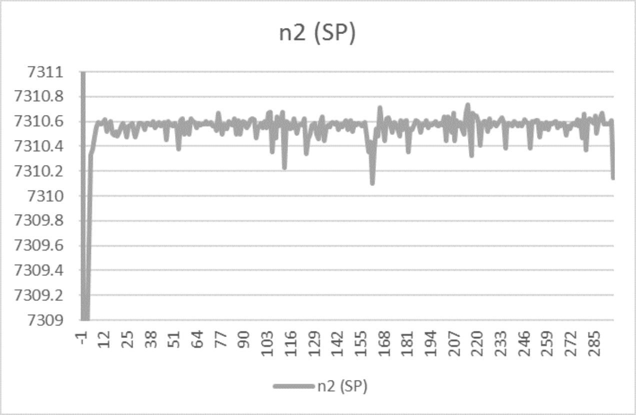
Perfect Foresight x Trend Chaser



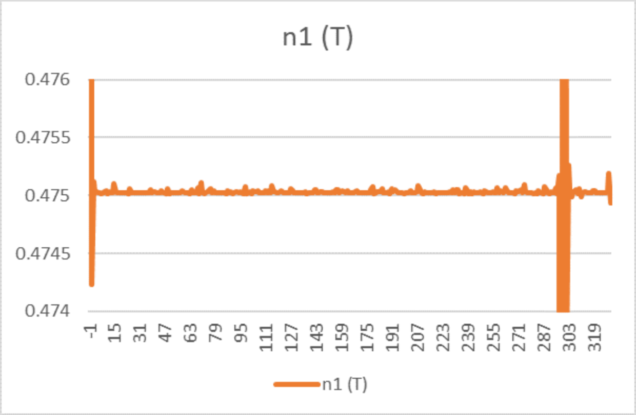
Fundamentalist x Trend Chaser



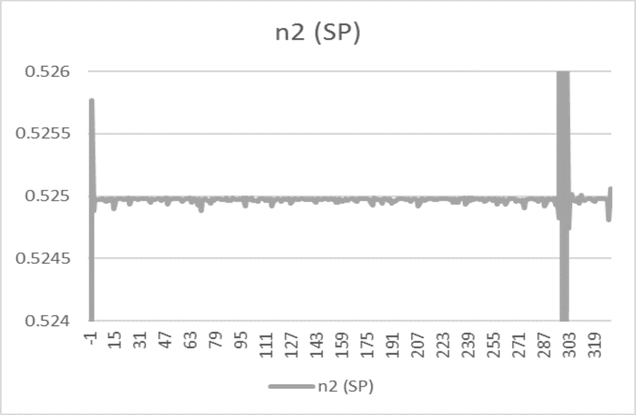
Fundamentalist x Trend Chaser



Fundamentalist x Contrarian



Fundamentalist x Contrarian



Future Work

- Extend the model to match real data
- Check fitnesses with CAPM
- Increase variables analysis
- Run simulations with different inputs for parameter values
- Run simulations with NIKKEI data
- Adjust levels of noise, compare to real economic shocks
- Analyze chaotic dynamics

References

Brock, Hommes (1998)
Heterogeneous beliefs and routes to chaos in a simple asset pricing model
Journal of Economic Dynamics and Control 22(1-2), 1235 – 1274

Friedman, Abraham (2009)
Bubbles and crashes: Gradient dynamics in financial markets
Journal of Economic Dynamics and Control 33(4), 922 – 937

LeBaron, Arthur and Palmer (1999)
Time series properties of an artificial stock market
Journal of Economic Dynamics and Control 23(9-10), 1487 – 1516

Questions?